XXI. *New Fundamental Experiments upon the Collision of Bodies. By Mr.* John Smeaton, *F. R. S. in a Letter to Sir* Joseph Banks, *Bart. P. R. S.*

Read April 18, 1782.

TO SIR JOSEPH BANKS, BART. P. R. S.

SIR,

THE subjects of the inclosed tract have been the object of my consideration for many years past; and as they contain some matters that have not only been variously reasoned about, but variously concluded upon; if what is contained therein shall appear of such a nature as either to establish truth, as it appears to me; or to prompt some more able person, in reviewing the subject, to shew what links in my chain of reasoning thereon are defective, so as to establish the whole doctrine of moving bodies upon one plain consistent basis, my end will be equally answered in offering them to you, to be laid before the Royal Society, in case you shall think that the importance of the subject shall merit the same: furthermore, I hope to be forgiven, if in some parts of this paper I have expressed myself with more pointedness than I might have done, for I declare, that it was solely owing to my earnestness that the subject of mechanic motions and powers should be fully and freely investigated, and established upon grounds that shall be uncontrovertible.

I have the honour to be, &c.

Gray's-Inn,
April 18, 1782.

IT is univerſally acknowledged, that the firſt ſimple principles of ſcience cannot be too critically examined, in order to their being firmly eſtabliſhed; more eſpecially thoſe which relate to the practical and operative parts of mechanics, upon which much of the active buſineſs of mankind depends. A ſentiment of this kind occaſioned my tract upon *Mechanic Power*, which was publiſhed in the Philoſophical Tranſactions, vol. LXVI. for the year 1776. What I have now to offer was intended as a ſupplement thereto, and the experiments were then, in part, tried; but the completion thereof was deferred at that time, partly from want of leiſure; partly to avoid too great a length of the paper itſelf; and partly to avoid the bringing forward too many points at once. My preſent purpoſe is to ſhew, that the true doctrine of the *colliſion of bodies* hangs as it were upon the ſame hook, as the doctrine of the gradual generation of motion from reſt, conſidered in that paper; that is, that whether bodies are put into gradual motion, and uniformly accelerated from reſt to any given velocity; or are put in motion, in an inſtantaneous manner, when bodies of any kind ſtrike one another; the motion, or ſum of the motions produced, has the ſame relation to mechanic power therein defined, which is neceſſary to produce the motion deſired. To prove this, and at the ſame time to ſhew ſome capital miſtakes in principle, which have been *aſſumed* as indiſputable truths by men of great learning, is the reaſon of my now purſuing the ſame ſubject.

I do not mean to point out the particular miſtakes which have been made by particular men, as that would lead me into too great a length: I ſhall therefore content myſelf with obſerving, that the laws of colliſion, which have been inveſtigated by mathematical philoſophers, are principally of three kinds; *viz.* thoſe relating to bodies perfectly *elaſtic*; to bodies

perfectly

perfectly unelastic, and perfectly *soft*; and to bodies perfectly unelastic, and perfectly *hard*. To avoid prolixity, I shall consider in each, only the simple case of two bodies which are equal in weight or quantity of matter striking one another. Respecting those which are perfectly elastic, it is universally agreed, that when two such bodies strike one another, no motion is lost; but that in all cases, what is lost by one is acquired by the other: and hence, that if an elastic body in motion strikes another at rest, upon the stroke the former will be reduced to a state of rest, and the latter will fly off with an equal velocity.

In like manner, if a non-elastic *soft* body strikes another at rest, they neither of them remain at rest, but proceed together from the point of collision with exactly one half of the velocity that the first had before the stroke; this is also universally allowed to be true, and is fully proved by every good experiment upon the subject.

Respecting the third species of body, that is, those that are non-elastic, and yet perfectly hard; the laws of motion relating to them, as laid down by one species of philosophers, have been rejected by another; the latter alledging, that there are no such bodies to be found in nature whereon to try the experiment; but those who have laid down and assigned the doctrine that would attend the collision of bodies of this kind (if they could be found) have universally agreed, that if a non-elastic *hard* body was to strike another of the same kind at rest, that, in the same manner as is agreed concerning non-elastic soft bodies, they neither of them would remain at rest, but would in like manner proceed from the point of collision, with exactly one half of the velocity that the first had before the stroke: in short, they lay it down as a rule attending all non-elastic bodies, whether hard or soft, that the velocity after the stroke will be the ſame in both, *viz. one half* of the velocity of the original ſtriking body.

Here is therefore the aſſumption of a principle, which in reality is proved by no experiment, nor by any fair deduction of reaſon that I know of, *viz.* that the velocity of non-elaſtic *hard* bodies after the ſtroke muſt be the ſame as that reſulting from the ſtroke of non-elaſtic *ſoft* bodies; and the queſtion now is, whether it is true or not?

Here it may be very properly aſked, what ill effects can reſult to practical men, if philoſophers ſhould reaſon wrong concerning the effects of what does not exiſt in nature, ſince the practical men can have no ſuch materials to work upon, or misjudge of? But it is anſwered, that they who infer an equality of effects between the two ſorts, may from thence be miſled themſelves, and in conſequence miſlead practical men in their reaſonings and concluſions concerning the ſort with which they have abundant concern, to wit, the non-elaſtic *ſoft bodies*, of which water is one, which they have much to do with in their daily practice.

Previous to the trying my experiment on mills I never had doubted the truth of the doctrine, that the ſame velocity reſulted from the ſtroke of both ſorts of non-elaſtic bodies; but the trial of thoſe experiments made me clearly ſee at leaſt the inconcluſiveneſs, if not the falſity of that doctrine: becauſe I found a reſult which I did not expect to have ariſen from either ſort; and for the which, when it appeared from experiment, I could ſee a ſubſtantial reaſon why it ſhould take place in one ſort, and that it was impoſſible that it could take place in the other; for if it did, the bodies could not have been perfectly *hard*, which would be contrary to the hypotheſis. Of this deduction I have given notice in my ſaid tract on mills, publiſhed

lished in the Philosophical Transactions, vol. LI. for the year 1759 *.

It may also be said, that since we have no bodies perfectly elastic, or perfectly unelastic and *soft*, why should we expect and bodies perfectly unelastic and *hard?* Why may not the effects be such as should result from a supposition of their being *imperfectly elastic* joined with their being *imperfectly hard?* But here I must observe, that the supposition appears to be a contradiction in terms.

We have bodies which are so nearly perfectly elastic, that the laws may be very well deduced and confirmed by them; and the same obtains with respect to non-elastic *soft* bodies; but concerning bodies of a mixed nature, which are by far the greatest number, so far as they are wanting in elasticity, they *are soft*, and *bruise*, *yield*, or *leave a mark* in collision; and so far as they are not perfectly soft they are elastic, and observe a mixture of the law relative to each; but imperfectly elastic bodies, imperfectly hard come in reality under the *same description* as the former mixed bodies: for so far as they are imperfectly hard they are soft, and either *bruise* and *yield*, or leave a mark in the stroke; and so far as they want perfect elasticity, they are non-elastic; that is to say, they are bodies imperfectly elastic, and imperfectly soft; and in fact I have never yet seen any bodies but what come under this description. It seems, therefore, that respecting the *hardness* of bodies they differ in degrees of it, in proportion as they have a greater degree of tenacity or cohesion; that is, are further removed from perfect

* "The effect, therefore, of overshot wheels, under the same circumstance of "quantity and fall, is at a medium double to that of the undershot: and as a "consequence thereof, that *non-elastic bodies, when acting by their impulse or collision,* "*communicate only a part of their original power*; the other part *being spent in* "*changing their figure in consequence of the stroke.*" Phil. Trans. vol. LI. p. 133.

softness,

ſoftneſs, at the ſame time that their elaſtic ſprings, ſo far as they reach, are very ſtiff; and hence we may (by the way) conclude, that the ſame mechanic power that is required to change the figure in a *ſmall degree* of thoſe bodies that have the popular appellation of *hard bodies*, would change it in a *great degree* in thoſe bodies that approach towards ſoftneſs, by having a ſmall degree of tenacity or coheſion. In the former kind we may rank the harder kinds of *caſt iron*, and in the latter, *ſoft tempered clay*.

While the philoſophical world was divided by the diſpute about the *old* and *new opinion*, as it was called, concerning the powers of bodies in motion, in proportion to their different velocities: thoſe who held the old opinion contending, that it was as the velocity *ſimply*, aſked thoſe of the new, How, upon their principles, they would get rid of the concluſions ariſing from the doctrine of unelaſtic perfectly hard bodies? They replied, They found no ſuch bodies in nature, and therefore did not concern themſelves about them. On the other hand, thoſe of the new opinion aſked thoſe of the old, How they would account for the caſe of non-elaſtic ſoft bodies, where, according to them, the whole motion loſt by the ſtriking body was retained in the two after the ſtroke (the two bodies moving together with the half velocity), though the two non-elaſtic bodies had been bruiſed and changed their figure by the ſtroke; for, if no motion was loſt, the change of figure muſt be an effect without a cauſe? To obviate this, thoſe of the old opinion ſeriouſly ſet about proving, that the bodies might change their figure, without any loſs of motion in either of the ſtriking bodies.

Neither of theſe anſwers have appeared to me ſatisfactory, eſpecially ſince my mill experiments: for with reſpect to the firſt, it is no proper argument to urge the impoſſibility of finding

ing the proper material for an experiment, in answer to a conclusion drawn from an abstract idea. On the other hand, if it can be shewn, that the figure of a body can be changed, without a *power*, then, by the same law, we might be able to make a *forge hammer* work upon a mass of soft iron, without any other power than that necessary to overcome the friction, resistance, and original *vis inertiæ*, of the parts of the machine to be put in motion: for, as no progressive motion is given the mass of iron by the hammer (it being supported by the anvil), no power can be expended that way; and if none is lost to the hammer from changing the figure of the iron, which is the only effect produced, then the whole power must reside in the hammer, and it would jump back again to the place from which it fell, just in the same manner as if it fell upon a body perfectly elastic, upon which, if it did fall, the case would really happen: the power, therefore, to work the hammer would be the same, whether it fell upon an elastic or non-elastic body; an idea so very contrary to all experience, and even apprehension, of both the philosopher and vulgar artist, that I shall here leave it to its own condemnation.

As nothing, however, is so convincing to the mind as experiments obvious to the senses, I was very desirous of contriving an experiment in point; and as I saw no hopes of finding matter to make a *direct* experiment, I turned my mind towards an indirect one; so circumscribed, however, as to prove incontestably, that the result of the stroke of two non-elastic perfectly hard bodies could not be the same as would result from the collision of two soft ones; that is, if it can be *bona fide* proved, that *one half* of the original power is lost in the stroke of soft bodies by the change of figure (as was very strongly suggested by the mill experiments); then since no such loss

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can happen in the collifion of bodies perfectly hard, the refult and confequence of fuch a ftroke muft be *different*.

The confequence of a ftroke of bodies perfectly hard, but void of elafticity, muft doubtlefs be different from that of bodies perfectly *elaftic*: for having no fpring the body at reft could not be driven off with the velocity of the ftriking body, for that is the confequence of the action of the fpring or elaftic parts between them, as will be fhewn in the refult of the experiments; the ftriking body will therefore not be ftopped, and as the motion it lofes muft be communicated to the other, from the equality of action and re-action, they will both proceed together, with an equal velocity, as in the cafe of non-elaftic foft bodies: the queftion, therefore, that remains is, what that *velocity muft be?*—It muft be greater than that of the non-elaftic foft bodies, becaufe there is no mechanical power loft in the ftroke. It muft be lefs than that of the ftriking body, becaufe, if equal, inftead of a *lofs* of motion by the collifion, it will be doubled. If, therefore, non-elaftic foft bodies lofe half their motion, or mechanical power, by change of figure in collifion, and yet proceed together with half the velocity, and the non-elaftic hard bodies can lofe *none* in any manner whatever; then, as they muft move together, their velocity muft be fuch as to preferve the equality of the mechanic power, *unimpaired*, after the ftroke the fame as it was before it.

For example, let the velocity of the ftriking body before the ftroke be 20, and its mafs or quantity of matter 8; then, according to the rule deduced from the experiments in the tract on *Mechanic Power* (fee exp. third and fourth) that power will be expreffed by $20 \times 20 = 400$, which $\times 8 = 3200$; and if half of it is loft in the ftroke, in the cafe of non-elaftic foft bodies, it will be reduced to 1600; which $\div$ 16 the double quantity of matter, will give 100 for the fquare of their velocity; the

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ſquare root of which being 10, will be the velocity of the two non-elaſtic ſoft bodies after the ſtroke, being juſt one half of the original velocity, as it is conſtantly found to be. But in the non-elaſtic hard bodies, no power being loſt in the ſtroke, the mechanic power will remain after it, as before it, = 3200; this, in like manner, being divided by 16, the double quantity of matter, will give 200 for the ſquare of the velocity, the ſquare root of which is 14. 14, &c. for their velocity after the ſtroke, which is to 10, the velocity of the non-elaſtic ſoft bodies after the ſtroke, as the ſquare root of 2 to 1; or as the diagonal of a ſquare to its ſide.

It remains, therefore, now to be proved, that preciſely half of the mechanic power is *loſt* in the colliſion of non-elaſtic ſoft bodies; for which purpoſe my mind ſuggeſted the following reflections. In the colliſion of elaſtic bodies the effect ſeemingly inſtantaneous, is yet performed in *time*; during which time the natural ſprings reſiding in elaſtic bodies, and which conſtitute them ſuch, are bent or forced, till the motion of the ſtriking body is divided between itſelf and the body at reſt; and in this ſtate the two bodies would then proceed together, as in the caſe of non-elaſtic ſoft bodies; but as the ſprings will immediately reſtore themſelves in an equal time, and with the ſame degree of *impulſive force*, wherewith they were bent in this re-action, the motion that remained in the ſtriking body will be totally deſtroyed, and the total exertion of the two ſprings, communicated to the original reſting body, will cauſe it to fly off with the ſame velocity wherewith it was ſtruck.

Upon this idea, if we could conſtruct a couple of bodies in ſuch a way that they ſhould either act as bodies perfectly elaſtic; or, that their ſprings ſhould at pleaſure be hooked up, retained, or prevented from reſtoring themſelves, when at their extreme degree of bending; and if the bodies under theſe circumſtances obſerved the laws of colliſion of non-elaſtic ſoft bodies, then it would be proved, that one half of the mechanical power, reſiding in the ſtriking body, would be loſt in the action of colliſion; becauſe the impulſive force or power of the ſpring in its reſtitution being cut off, or ſuſpended from acting, which is equal to the impulſive force or power to bend it (and which alone has been employed to communicate motion from one body to the other), it would make it evident, that one half of the impulſive force is loſt in the action, as the other half remains *locked up* in the ſprings. It alſo follows, as a *collateral circumſtance*, that be the impulſive power of the ſprings what it may from firſt to laſt, yet as one half of the *time* of the action is by this means cut off, in this ſenſe alſo it will follow, that one half of the mechanic power is deſtroyed; or rather, in this caſe, remains locked up in the ſprings, capable of being *re-exerted* whenever they are ſet at liberty, and of producing a freſh mechanical effect, equivalent to the motion or mechanical power of the two non-elaſtic ſoft bodies after their colliſion.

Hence we muſt infer, that the quantity of mechanical power expended in diſplacing the parts of non-elaſtic ſoft bodies in colliſion, is exactly the ſame as that expended in bending the ſprings of perfectly elaſtic bodies; but the difference in the ultimate effect is, that in the non-elaſtic ſoft bodies, the power taken to diſplace the parts will be totally loſt and deſtroyed, as it would require an equal mechanic power to be raiſed a-freſh, and exerted in a contrary direction to reſtore the parts back again to their former places; whereas, in the caſe of the elaſtic bodies, the operation of half the mechanic power is, as obſerved already, only locked up and ſuſpended, and capable of being re-exerted without a further original acceſſion.

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Thefe ideas arofe from the refult of the experiments tried upon the machine defcribed in my faid tract upon Mechanic Power, and were alfo communicated to my very worthy and ingenious friend WILLIAM RUSSELL, Efq. F. R. S. at the fame time that I fhewed him thofe experiments in 1759; but the mode of putting this matter to a full and fair mechanical trial has fince occurred; and though fome rough trials, fufficient to fhew the effect, were made thereon, prior to the offering the paper on mechanical power to the Society in 1776, yet the machine itfelf I had not leifure to complete to my fatisfaction till lately; which I mention to apologize for the length of time that thefe fpeculations have taken in bringing forward.

DESCRIPTION OF THE MACHINE FOR COLLISION.

Fig. 1. fhews the front of the machine as it appears at reft when fitted for ufe.

A is the pedeftal, and AB the pillar, which fupports the whole, C, D are two compound bodies of about a pound weight each, but as nearly equal in weight as may be. Thefe bodies are alike in conftruction, which will be more particularly explained by fig. 2. Thefe bodies are fufpended by two white fir rods of about half an inch diameter *ef* and *gh*, being about four feet long from the point of fufpenfion to the center of the bodies; and their fufpenfion is upon the crofs-piece II, which is mortoifed through, to let the rods pafs with perfect freedom; and they hang upon two fmall plates filed to an edge on the under fide, and pafs through the upper part of the rods. Their centers are at *k* and *l*, and the edges being let into a little notch, on each fide the mortoife, the rods are at liberty to vibrate freely upon their refpective points (or rather edges) of fufpenfion, and are determined to one plain of vibration. MN is a flat arch of white wood, which may be covered with

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Paper, that the marks thereupon may be the more confpicuous.

The crofs-piece II is made to project fo far before the pillar, that the bodies in their vibrations may pafs clear of it, without danger of ftriking it; and alfo the arch MN is brought fo far forward as to leave no more than a clearance, fufficient for the rods to vibrate freely without touching it.

Fig. 2. Shews one of the compound bodies, drawn of its full fize. AB is a block of wood, and about as much in breadth as it is reprefented in height, through a hole in which the wood rod CC paffes, and is fixed therein.

DB reprefents a plate of lead about three-eighths of an inch thick, one on each fide, fcrewed on by way of giving it a competent weight. *dBefg* reprefents the edge of a fpringing plate of brafs, rendered elaftic by hard hammering; it is about five-eighths of an inch in breadth, and about one-twentieth of an inch thick. It is fixed down upon the wooden block at its end *dB* by means of a bridge plate, whofe end is fhewn *hi*, and is fcrewed down on each fide the fpring plate by a fcrews which being relaxed the fpring can be taken out at pleafure, and adjufted to its proper fituation. *kl* is a light thin flip of a plate, whofe under edge is cut into teeth like a fine faw or ratchet, and is attached to the fpring by a pin at *k*, which paffes through it, and alfo through a fmall ftud rivetted into the back part of the fpring, and upon which pin, as a center, it is freely moveable.

mn fhews a fmall plate or ftud feen edgeways raifed upon the bridge plate, through an hole in which ftud the ratchet paffes; and the lower part of the hole is cut to a tooth fhaped properly to catch the teeth of the ratchet, and retain it together with the fpring at any degree to which it may be fuddenly bent; and for this intent it is kept bearing gently downward, by means of a wire-fpring *opq*, which is in reality double, the

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bearing part at *o* being semi-circular; from which branching off on each side the rod *cc*, passes to *p*, and fixes at each end into the wood at *q*. However, to clear the ratchet, which is necessarily in the middle as well as the rod, the latter is perforated; and also the block is cut away, so far as to set the main spring at *e* free of all obstacles that would prevent its play from the point B. The part *fg* is shewn thicker than the rest, by being covered with thin kid leather tight sowed on, to prevent a certain jarring that otherwise takes place on the meeting of the springs in collision.

Let us now return to fig. 1. the marks upon the arch MN are put on as follows. *op* is an arch of a circle from the center *l*, and *qr* an arch of a circle from the center *k* intersecting each other at *s*. Now the middle line of the marks *t*, *v*, are at the same distance from the middle line at *s* that the centers *kl* are; so that when each body hangs in its own free position, without bearing against the other, the rod *ef* will cover the mark at *t*, and the rod *gh* will cover the mark at *v*. From the point S upon the arches S*p* and S*q* respectively, set off points at an equal and competent distance from S each way, which will give the middle of the mark *w* and *x*: and upon the arch S*p* find a middle point between the mark *v* and *w*, which let be *y*; and on the other side, in like manner, upon the arch S*q* find a middle point for the mark *z*; then set off the distance S*v* or S*t* from *y* each way, and from *z* each way; and from these points, drawing lines to the respective centers *l* and *k*, they will give the place and position of the marks *a*, *b*, and *c*, *d*; and thus is the machine prepared for use.

FOR TRIALS ON ELASTIC BODIES.

For this use take out the pins and ratchets from each respectively, and the springs being then at liberty, with a short bit of ſtick (ſuppoſe the ſame ſize as the rods) turn aſide the rod *gh* with the right-hand, carrying the body D upwards till the ſtick is upon the mark *w*, as ſuppoſe at ⊙; there hold it, and with the left ſet the body C perfectly at reſt; in which caſe the rod *ef* will be over the mark *t*; then ſuddenly withdraw the ſtick, in the direction that the rod *gh* is to follow it, and the ſpring of the body D, impinging upon that of the body C, they will be both bent, and alſo reſtored; and the body C will fly off, and mount till its rod *ef* covers the mark *x*; the rod of the ſtriking body D remaining at reſt upon its proper mark of reſt *v*, till the body C returns, when the body D will fly off in he ſame manner; the two bodies thus rebounding a number of times, loſing a part of their vibration each time; but ſo nearly is the theory of elaſtic bodies fulfilled hereby, that the ſingle advantage of originally puſhing the rod *gh* beyond the mark *w*, by the thickneſs of the ſtick, or its own thickneſs, is ſufficient to carry the rod of the quieſcent body C completely to its mark *x*.

There are ſeveral other experiments that may be made with this apparatus, in confirmation of the doctrine of the colliſion of elaſtic bodies; which being univerſally agreed upon, and well known, it is needleſs further to dwell upon here; but reſpecting the application to non-elaſtic ſoft bodies, it is far more difficult to come at a fitneſs of materials for this kind of experiments, than it is for thoſe ſuppoſing perfectly elaſticity. The concluſions, however, may be attained with equal certainty.

FOR TRIALS ON NON-ELASTIC SOFT BODIES.

For this purpoſe the ratchets muſt be applied and put in order as before deſcribed, and the ſprings being both put to their point of reſt, let the body D be put to its mark *w* in the ſame manner as before deſcribed, and the body C to reſt. The body D being let go, and ſtriking the body C at reſt, in conſequence of the ſtroke, the ſprings being hooked up by the ratchets, they both move from their reſting marks *t*, *v*, reſpectively toward M: Now if they both moved together, and the rod *ef* covered the mark *c*, and the rod *gh* covered the mark *d* at their utmoſt limit, then they would truly obey the laws of non-elaſtic ſoft bodies; becauſe their medium aſcent would be to the mark *z*, which is juſt half the angle of aſcent to the mark *x*; but as in this piece of machinery, though the main or principle ſprings are hooked up, yet every part of them, and all the materials of which they are compoſed, and to which they are attached, have a degree, or more properly ſpeaking, a certain compaſs of *elaſticity*, which, as ſuch, is perfect, and no motion loſt thereby.

We muſt not, therefore, expect the two compound bodies after the ſtroke to ſtick together without ſeparating, as would be the caſe with bodies truly non-elaſtic and ſoft; but that from the elaſticity they are poſſeſſed of, they will by rebounding be ſeparated; but that elaſticity being perfect, can occaſion no loſs of motion to the ſum of the two bodies; ſo that if the body C aſcends as much above its mark *c* as the body D falls ſhort of its mark *d*, then it will follow, that their medium aſcent will ſtill be to the mark *z*, as it ought to have been, had theybeen truly non-elaſtic ſoft bodies; and this, in reality, is truly the caſe in the experiment, as nearly as it can be diſcerned.

After a few vibrations, by the rubbing of the ſprings againſt one another, they are ſoon brought to reſt; and here they would *always reſt* had they been truly and properly perfect non-elaſtic ſoft bodies; but here, as in the caſe of theſe bodies, by a change of the figure and ſituation of the component parts, there is expended one half of the mechanical power

of

of the firft mover, yet in this cafe the other half is not *loft*, but *fufpended*, ready to be re-exerted whenever it is fet at liberty; and that it is really and *bona fide one half* and neither more or lefs, appears from this uncontroverted fimple principle, that the power of reftitution of a perfect fpring is exactly equal to the power that bends it. And this may, in a certain degree, be fhewn to be fact by experiment, if there were any need of fuch a proof; for if, when the bodies are at reft after the laft experiment, the two rods are lafhed together near the bottom with a bit of thread, and then the ratchets unpinned and removed; on cutting the thread with a pair of fciffars they will each of them rebound, C towards M, and D towards N; and if they rebounded refpectively to *z* and *y*, the mechanical power exerted would be the fame as it was after the ftroke, when the mean of their two afcents was up to the mark *z*; but here it is not to be expected, becaufe not only the motion loft by the friction of the ratchets is to be deducted, becaufe it had the effect of real non-elafticity; but alfo the elafticity that feparated them in the ftroke, which was loft in the vibrations that fucceeded; neither of which hindered the mean afcent to be to *z*; but yet, under all thefe difadvantages in the machine (if not unreafonably ill made) the rod *ef* will afcend to *d*, and *gh* to *a*: and hence I infer, as a pofitive truth, that in the collifion of non elaftic foft bodies, *one half of the mechanic power refiding in the ftriking body is loft in the ftroke.*

Refpecting bodies unelaftic and perfectly hard, we muft infer, that fince we are unavoidably led to a conclufion concerning them, which contradicts what is efteemed a truth capable of the ftricteft demonftration; *viz.* that the velocity of the center of gravity of no fyftem of bodies can be changed by any collifion betwixt one another, fomething muft be affumed that involves a contradiction. This perfectly holds,

 according

according to all the eſtabliſhed rules, both of perfectly elaſtic and perfectly non-elaſtic *ſoft* bodies; rules which muſt fail in the perfectly non-elaſtic *hard* bodies, if their velocity after the ſtroke is to the velocity of the ſtriking body as one is to the ſquare root of 2; for then the center of gravity of the two bodies will by the ſtroke acquire a velocity greater than the center of gravity the two bodies had before the ſtroke in that proportion, which is proved thus.

At the outſet of the ſtriking body, the center of gravity of the two bodies in our caſe will be exactly in the middle between the two; and when they meet it will have moved from their half diſtance to their point of contact, ſo the velocity of the center of gravity before the bodies meet will be exactly one half of the velocity of the ſtriking body; and, therefore, if the velocity of the ſtriking body is 2, the velocity of the center of gravity of both will be one. After the ſtroke, as both bodies are ſuppoſed to move in contact, the velocity of the center of gravity will be the ſame as that of the bodies; and as their velocity is proved to be the ſquare root of 2, the velocity of their center of gravity will be increaſed from 1. to the ſquare root of 2.; that is, from 1. to 1.414, &c.

The fair inference from theſe contradictory concluſions therefore is, that an unelaſtic hard body (perfectly ſo) is a repugnant idea, and contains in itſelf a contradiction; for to make it agree with the fair concluſions that may be drawn on each ſide, from clear premiſes, we ſhall be obliged to define its properties thus: that in the ſtroke of unelaſtic hard bodies they cannot *poſſibly loſe any* mechanic power in the ſtroke; becauſe no other impreſſion is made than the communication of motion; and yet they *muſt loſe* a *quantity* of mechanic power in the ſtroke; becauſe, if they do not, their common center of gravity, as

 above

above ſhewn, will acquire an *increaſe* of velocity by their ſtroke upon each other.

In a like manner the idea of a *perpetual motion*, perhaps, at firſt ſight, may not appear to involve a contradiction in terms; but we ſhall be obliged to confeſs that it does, when, on examining its requiſites for execution, we find we ſhall want bodies having the following properties; that when they are made to *aſcend* againſt gravitation their abſolute weight ſhall be *leſs*; and that when they *deſcend* by gravitation (through an equal ſpace) their abſolute weight ſhall be greater; which, according to all we know of nature, is a *repugnant* or *contradictory idea*.

Machine for COLLIS

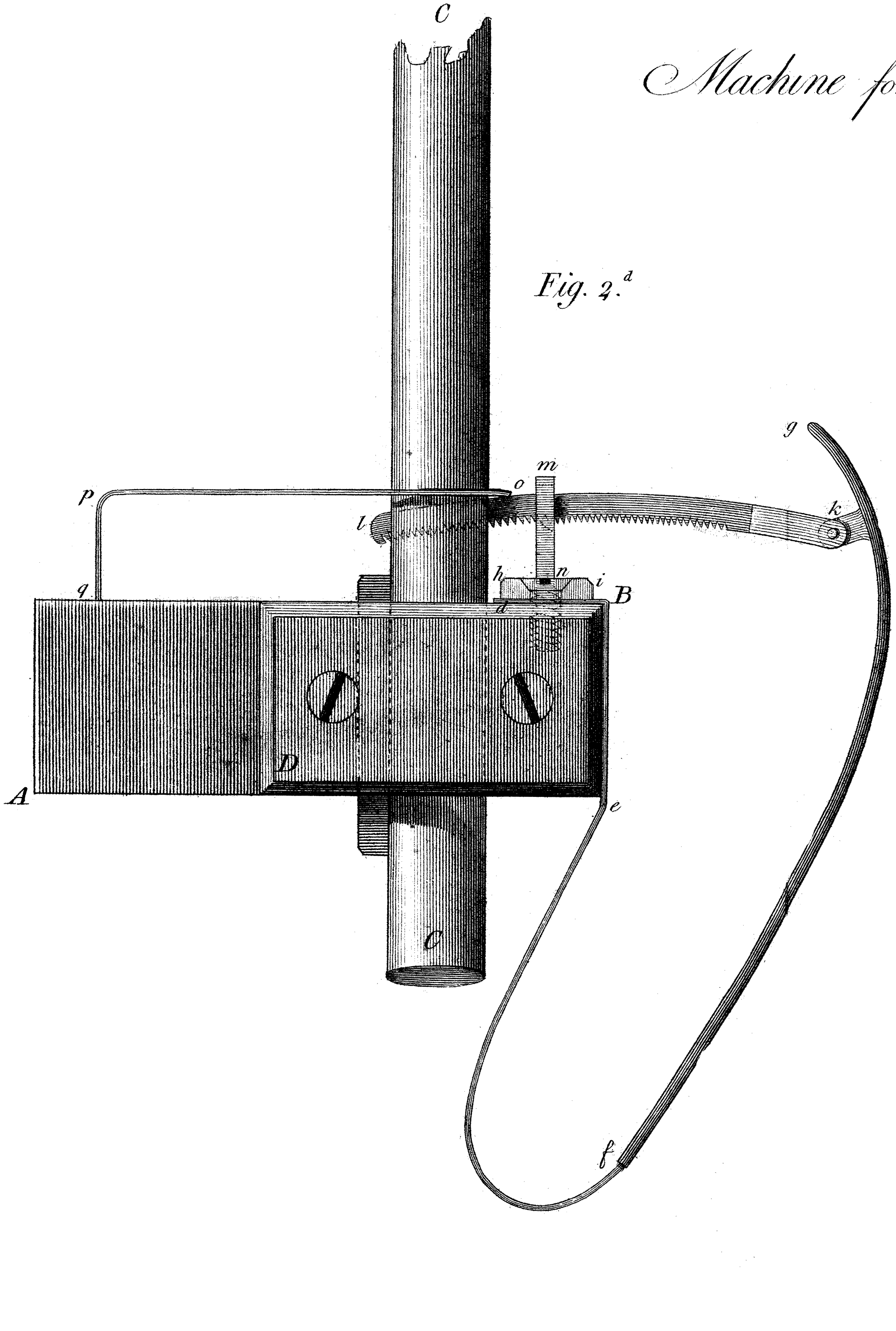
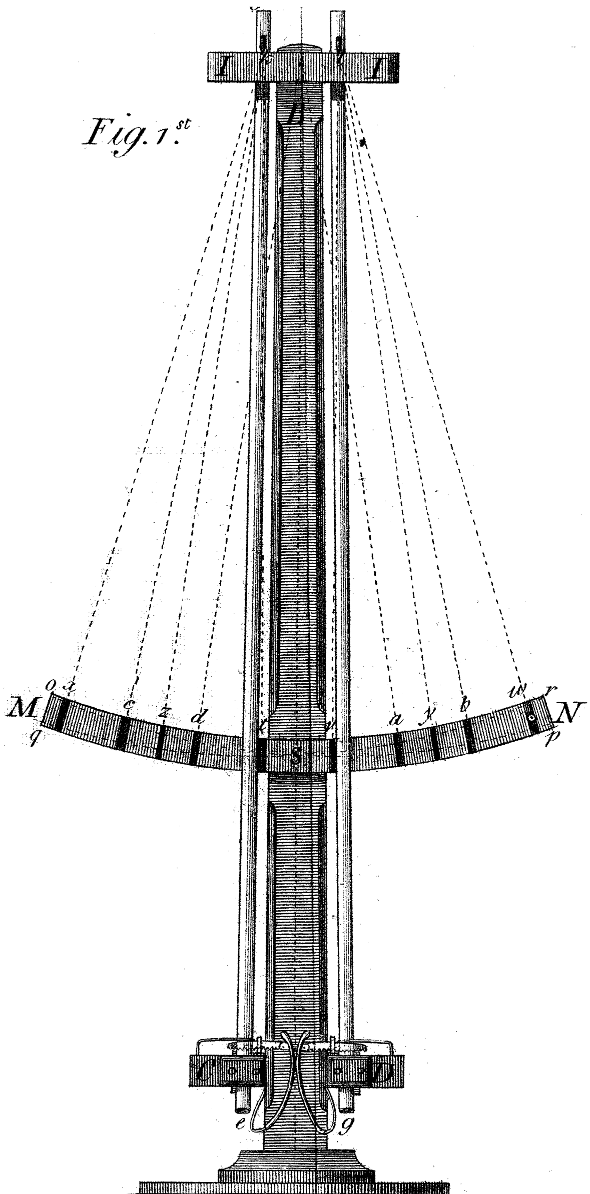


Fig. 2.[d]

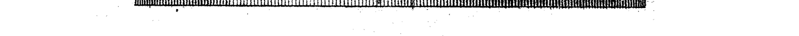
12 In.

LLISION.

Fig. 1st



Scale of Feet.



Machine for COLLISION

Fig. 2.d

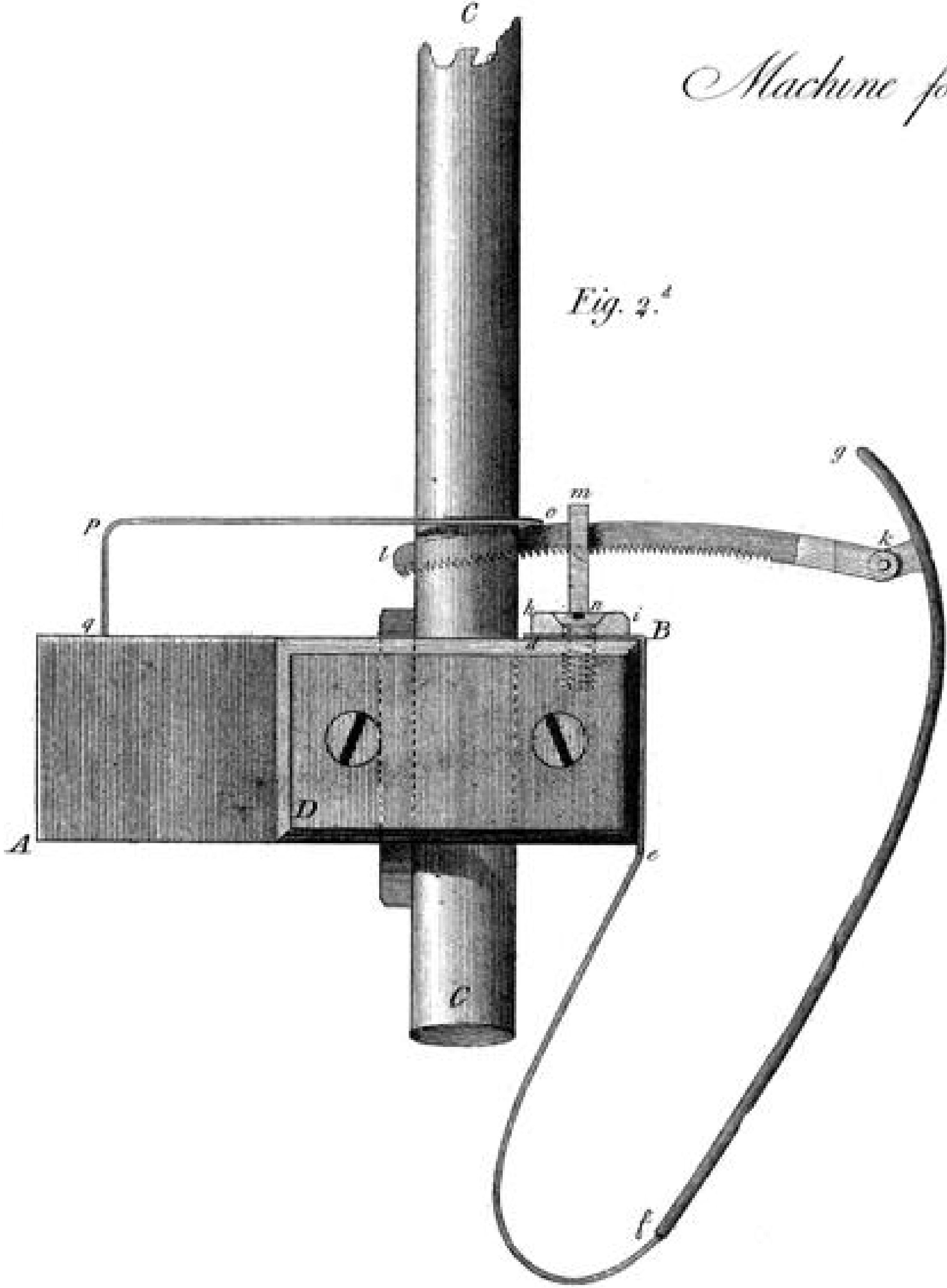
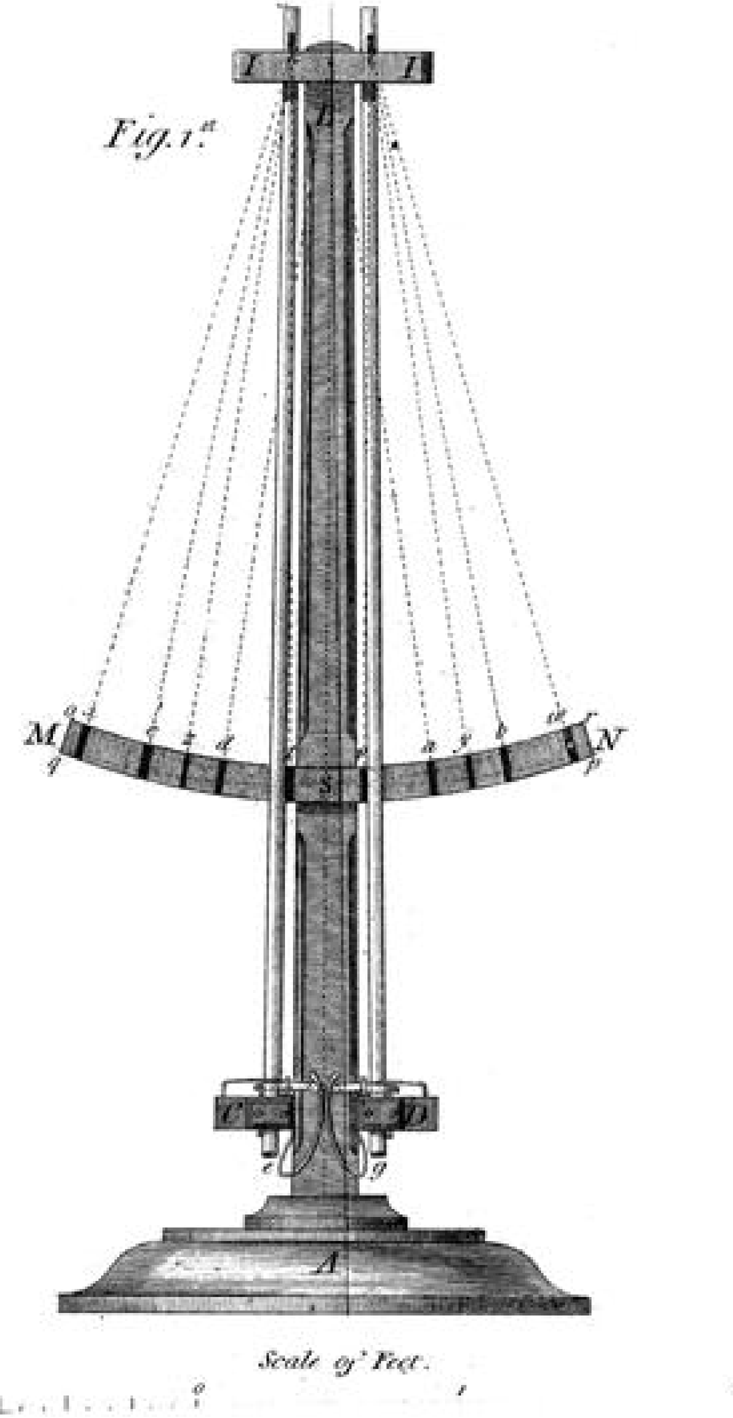


Fig. 1.st



Scale of Feet.